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Infra-red soft universality

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We show that in a special class of theories the commonly assumed universal form of the soft supersymmetry-breaking terms is approached in the infra-red limit. The resulting universal scalar mass and trilinear coupling are predicted in terms of the gaugino mass.

The supersymmetric standard model (SSM) has gained widespread acceptance as a framework for physics at and above 1 TeV, encouraged by the unification of gauge couplings within this theory at a scale of $M_U = 10^{16}$ GeV. It is desirable that the soft breaking couplings should adopt a roughly “universal” form at M_U ; significant deviations from universality would give rise to flavour-changing neutral currents in the theory at the weak scale. It happens that popular scenarios which explain the soft-breaking terms as generated by supersymmetry breaking in the “hidden sector” of an underlying supergravity theory (possibly ultimately arising from string theory) do in fact make at least plausible a universal form of the kind required. However, this universal form would pertain at or near the Planck scale (M_P) and in general diversions away from universality would be expected as the couplings evolved down to M_U [1]. Although this need not be disastrous for phenomenology, it does mean that low energy predictions are sensitive to the nature of the unified theory; and the problem of a fully convincing explanation of the origin of universality at M_P remains unsolved.

In a recent paper [2] we showed that if the dimensionless couplings obeyed a certain relation (which we shall call generically the $P = \frac{1}{3}Q$ condition), then a particular universal form for the soft-breaking couplings was preserved by the renormalisation group evolution down to M_U . Moreover, this universal form is in fact predicted by a fairly generic superstring scenario in which supersymmetry breaking is engendered by dilaton and “size modulus” vevs. This is all well and good, but suffers from the drawback that there is no obvious reason for string theory to yield dimensionless couplings satisfying the required $P = \frac{1}{3}Q$ constraint. A much more interesting hypothesis, it seems to us, is the following: if the grand unified theory above M_U is such that dimensionless couplings can in principle be found to satisfy the $P = \frac{1}{3}Q$ condition, then such a configuration of dimensionless couplings may represent an attractive infra-red (IR) fixed point, which can be approached quite closely as the theory evolves towards M_U . Moreover the RG-invariant universal form for the soft-breaking couplings alluded to above may also then constitute an IR fixed point which again may be approached quite closely at M_U . This would mean that we would no longer have to impose universality at M_U , or M_P ; for a wide range of possible input parameters the unified theory would evolve towards universality at M_U . We have pursued the phenomenological consequences of this idea elsewhere [3][4]; in this paper we explore the conditions in which it may be realised at M_U using a number of toy models.

One might be tempted to argue that there is not a sufficient range of energy between the Planck scale, M_P , and M_U for any significant progress towards a fixed point to take

place. However, as was recently pointed out by Lanzagorta and Ross[5], the rate of evolution of couplings in the unified theory is enhanced relative to the SSM by the larger field content and by the potential lack of asymptotic freedom. Of course how precisely universality is approached will depend on the model.

We start by reviewing our previous work and then demonstrate the attractive nature of our fixed points in a restricted case. We then discuss in detail a rather more realistic $(SU_3)^3$ grand unified model which admits the $P = \frac{1}{3}Q$ condition and present some numerical results which display the approach to universality at M_U . We begin with our results for a general theory. The Lagrangian $L_{\text{SUSY}}(W)$ is defined by the superpotential

$$W = \frac{1}{6}Y^{ijk}\Phi_i\Phi_j\Phi_k + \frac{1}{2}\mu^{ij}\Phi_i\Phi_j. \quad (1)$$

L_{SUSY} is the Lagrangian for the $N = 1$ supersymmetric gauge theory, containing the gauge multiplet (λ being the gaugino) and a chiral superfield Φ_i with component fields $\{\phi_i, \psi_i\}$ transforming as a representation R of the gauge group \mathcal{G} . We assume that there are no gauge-singlet fields and that \mathcal{G} is simple. (The generalisation to a semi-simple group is trivial.) The soft breaking is incorporated in L_{SB} , given by

$$L_{\text{SB}} = (m^2)_i^j \phi^i \phi_j + \left(\frac{1}{6}h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2}b^{ij} \phi_i \phi_j + \frac{1}{2}M\lambda\lambda + \text{h.c.} \right) \quad (2)$$

(Here and elsewhere, quantities with superscripts are complex conjugates of those with subscripts; thus $\phi^i \equiv (\phi_i)^*$.) The superpotential W undergoes no infinite renormalisation so that we have, for instance

$$\beta_Y^{ijk} = Y^{ijp} \gamma^k_p + (k \leftrightarrow i) + (k \leftrightarrow j), \quad (3)$$

where γ is the anomalous dimension for Φ . The one-loop results for the gauge coupling β -function β_g and for γ are given by

$$16\pi^2 \beta_g^{(1)} = g^3 Q, \quad \text{and} \quad 16\pi^2 \gamma^{(1)i}_j = P^i_j, \quad (4)$$

where

$$Q = T(R) - 3C(G), \quad \text{and} \quad (5a)$$

$$P^i_j = \frac{1}{2}Y^{ikl}Y_{jkl} - 2g^2 C(R)^i_j. \quad (5b)$$

Here

$$T(R)\delta_{AB} = \text{Tr}(R_A R_B), \quad C(G)\delta_{AB} = f_{ACD}f_{BCD} \quad \text{and} \quad C(R)^i_j = (R_A R_A)^i_j. \quad (6)$$

The one-loop β -functions for the soft-breaking couplings are given by

$$16\pi^2 \beta_h^{(1)ijk} = h^{ijl} P^k_l + Y^{ijl} X^k_l + (k \leftrightarrow i) + (k \leftrightarrow j) \quad (7a)$$

$$16\pi^2 [\beta_{m^2}^{(1)}]_i^j = \frac{1}{2} Y_{ipq} Y^{pqn} (m^2)^j_n + \frac{1}{2} Y^{jpq} Y_{pqn} (m^2)^n_i + 2Y_{ipq} Y^{jpr} (m^2)^q_r + h_{ipq} h^{jpq} - 8g^2 M M^* C(R)^j_i, \quad (7b)$$

$$16\pi^2 \beta_b^{(1)ij} = b^{il} P^j_l + \mu^{il} X^j_l + (i \leftrightarrow j), \quad (7c)$$

$$16\pi^2 \beta_M^{(1)} = 2g^2 Q M, \quad (7d)$$

where

$$X^i_j = h^{ikl} Y_{jkl} + 4M g^2 C(R)^i_j \quad (8)$$

and we have dropped a $\text{tr}[R_A m^2]$ term in Eq. (7b) because \mathcal{G} is simple, and terms of the type $Y_{ijk} b^{jk}$ and $Y_{ijk} \mu^{jk}$ because there are no gauge singlets. We then showed in Ref. [2] that the conditions

$$h^{ijk} = -M Y^{ijk}, \quad (9a)$$

$$(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j, \quad (9b)$$

$$b^{ij} = -\frac{2}{3} M \mu^{ij} \quad (9c)$$

are RG invariant at one loop provided we impose the $P = \frac{1}{3}Q$ condition

$$P^i_j = g^2 P \delta^i_j = \frac{1}{3} g^2 Q \delta^i_j. \quad (10)$$

Moreover, the condition Eq. (10) is itself RG invariant up to at least two loops. In other words, dimensionless couplings satisfying Eq. (10) and soft-breaking couplings satisfying Eq. (9) represent fixed points of the RG evolution; it remains to confirm our claim that they can be IR attractive.[†] We shall do this analytically in a somewhat restricted case but our numerical experience with a more complex example indicates that this property, while

[†] In the special case of a finite theory, we have $P = Q = 0$, and soft breakings satisfying Eq. (9) preserve finiteness [6] [7]. For the $N = 4$ case, the fact that these results for the soft terms are approached in the IR limit was pointed out in Ref. [8].

not completely general, is at least a plausible feature of a realistic theory. Consider then the case of a theory with fields ϕ^i in an irreducible representation of \mathcal{G} , for which

$$C(R)^i{}_j = C(R)\delta^i{}_j, \quad Y^{ikl}Y_{jkl} = Y\delta^i{}_j \quad (11)$$

so that $P^i{}_j = g^2 P \delta^i{}_j$, where $P = Y/(2g^2) - 2C(R)$. It is easy to show that the standard fixed point[9] in the evolution of the ratio of the Yukawa to gauge couplings corresponds to $P = \frac{1}{3}Q$, and that it exists as long as $Q + 6C(R) > 0$.

Suppose further that we have soft-breaking couplings given by

$$h^{ijk} = -xMY^{ijk}, \quad (12a)$$

$$(m^2)^i{}_j = yMM^*\delta^i{}_j. \quad (12b)$$

It is easy to show using Eqs. (7)-(8) and Eq.(11) that at the fixed point $P = \frac{1}{3}Q$ we have

$$16\pi^2\beta_x = 12(x-1)C(R)g^2, \quad (13)$$

so that $x = 1$ is an IR fixed point. Then with $P = \frac{1}{3}Q$ and $x = 1$,

$$16\pi^2\beta_y = 2(y - \frac{1}{3})[6C(R) - Q]g^2 \quad (14)$$

so that $y = \frac{1}{3}$ is also an IR fixed point, as long as $6C(R) - Q > 0$. Finally if we suppose that the representation R also permits a quadratic invariant and set

$$b^{ij} = -\frac{2}{3}zM_\mu^{ij}, \quad (15)$$

we find that $z = 1$ is IR-attractive as long as $Q < 0$.

In more complicated cases, it can happen that while there does exist an IR-attractive fixed point for the dimensionless couplings, it does not correspond to $P = \frac{1}{3}Q$. The trilinear scalars will then still have the fixed point corresponding to Eq. (9a). This fixed point may or may not be IR attractive, however. In these cases neither Eq. (9b) nor Eq. (9c) will correspond in general to fixed points. Although the scalar mass evolution may still exhibit fixed point behaviour, this will not correspond to a *common* mass, as we have in Eq. (9b). It is for this reason that we favour theories which can satisfy $P = \frac{1}{3}Q$. It may also be that given a theory admitting $P = \frac{1}{3}Q$, the behaviour of the Yukawa couplings may be governed (for large initial values at M_P) by quasi-fixed point[5][10] rather than fixed-point behaviour. We should emphasise that in order to realise our goal of soft universality the

Yukawa couplings must approach the actual $P = \frac{1}{3}Q$ fixed point. It can also happen that while $P = \frac{1}{3}Q$ is indeed IR attractive, one or more of the conditions in Eq. (9) are saddle points. In specific models where this is the case, it can happen that for quite reasonable regions of parameter space the RG trajectories approach quite close to the saddle point when integrated from M_P to M_U . We will see an example of this later.

As our semi-realistic model we will take $SU_3 \otimes SU_3 \otimes SU_3$. $(SU_3)^3$ is a maximal subgroup of E_6 ; both groups have attractive features as candidate GUTs, particularly in the string context. Here we consider the basic case of a $(SU_3)^3$ theory with n sets each of the representations $X \equiv (3, 3, 1)$, $Y \equiv (1, \bar{3}, 3)$ and $Z \equiv (\bar{3}, 1, \bar{3})$. The superpotential for the theory is :

$$W = \frac{1}{3!}(\lambda_1 X^3 + \lambda_2 Y^3 + \lambda_3 Z^3) + \rho XYZ. \quad (16)$$

Here $\lambda_1 X^3 \equiv (\lambda_1)^{\alpha\beta\gamma} X_\alpha X_\beta X_\gamma$, where $\alpha, \beta \dots = 1 \dots n$. If we set the three gauge couplings all equal to g then it is easy to see that they remain equal under renormalisation, and $Q = 3n - 9$. (We may choose to imagine other sectors of the theory also contributing to Q , in which case Q becomes a free parameter, subject only to $Q > 3n - 9$.) The $P = \frac{1}{3}Q$ condition for this theory consists of the set of equations:

$$(2\lambda_i^2 + 3\rho^2)_\beta^\alpha = \frac{1}{3}(16 + Q)g^2\delta_\beta^\alpha, \quad i = 1 \dots 3 \quad (\text{no sum on } i). \quad (17)$$

where $(\lambda_i^2)_\beta^\alpha = (\lambda_i)^{\alpha\gamma\delta}(\lambda_i^*)_{\beta\gamma\delta}$. It is easy to see that these conditions are identical to those obtained by requiring the Yukawa couplings to be at the PR fixed points. Notice that in this case the Yukawa couplings are not completely determined by the $P = \frac{1}{3}Q$ condition.

In what follows we will suppose that we have $(\lambda_i^2)_\beta^\alpha = (\lambda_i^2)\delta_\beta^\alpha$, and $(\rho^2)_\beta^\alpha = \rho^2\delta_\beta^\alpha$. Assuming also that the soft ϕ^3 terms have the form $(h_i)^{\alpha\gamma\delta} = A_i(\lambda_i)^{\alpha\gamma\delta}$, and $(h_\rho)^{\alpha\gamma\delta} = A_\rho\rho^{\alpha\gamma\delta}$, then the fixed point conditions for the A -parameters are as follows:

$$(6\tilde{\lambda}_i^2 - Q)\tilde{A}_i + 6\tilde{A}_\rho\tilde{\rho}^2 + 16 = 0 \quad i = 1 \dots 3, \quad (18a)$$

$$\sum_i 2\tilde{A}_i\tilde{\lambda}_i^2 + (9\tilde{\rho}^2 - Q)\tilde{A}_\rho + 16 = 0, \quad (18b)$$

where we have defined $\tilde{\lambda}_i = \lambda_i/g$, $\tilde{\rho} = \rho/g$ and $\tilde{A}_i = A_i/M$. Imposing the $P = \frac{1}{3}Q$ condition we find the unique fixed point $\tilde{A}_i = \tilde{\rho} = -1$, corresponding, of course, to Eq. (9a). The stability matrix for the four \tilde{A}_i couplings has eigenvalues $16 - 9\tilde{\rho}^2$ (twice), 16, and $-Q$. Thus for IR stability we require $Q < 0$. This case is not favourable, even as a toy model, however; for example with $Q = -3$ the $P = \frac{1}{3}Q$ point is not approached very

rapidly. Turning to the case $Q > 0$, it is interesting that the eigenvector corresponding to the eigenvalue +16 is $(\tilde{A}_i, \tilde{A}_\rho) = (1, 1, 1, 1)$; this means that if we start at M_P with $\tilde{A}_i = \tilde{A}_\rho$, (or close to it) then we will be close to our fixed point $\tilde{A}_i = \tilde{A}_\rho = -1$ at M_U . Thus if string theory indeed dictates a universal A-parameter, then even though the fixed point corresponding to Eq. (9a) is a saddle point, it will still be IR attractive.

The fixed point for the soft $\phi\phi^*$ masses corresponding to Eq. (9b) has a stability matrix with eigenvalues $32 - 2Q - 18\tilde{\rho}^2 (= 12\lambda_i^2)$ (twice) and $32 - 2Q$, when $\lambda_{1\dots 3}$, ρ , and the A -parameters are at the fixed point. Thus we might expect good approach to universality for comparatively small $\tilde{\rho}$.

We now present some numerical results. In our analysis we run couplings and masses from M_P to M_U^\dagger and look for regions of parameter space such that the various soft parameters approach their fixed point values with a given degree of accuracy. We use $Q = 1$ and $g(M_U) = 0.72$ throughout; with this value the dimensionless couplings approach the $P = \frac{1}{3}Q$ point for a reasonable range of starting values. This behaviour is illustrated in Fig.1, where $P - \frac{1}{3}Q$ is plotted against energy scale for various starting values of the couplings at M_P . For all the curves we have $\lambda_1 = \lambda_2 = \lambda_3$, and $\lambda_i(M_P) = 5\rho(M_P)$.

Turning to the soft parameters, let us consider first what happens if we begin with “weak” universality at M_P (meaning values for the scalar masses and A -parameters that are universal, but not at the $P = \frac{1}{3}Q$ values). We use parameters x and y as defined in Eq. (12). In Fig.2 we show how the A -parameters converge, and it can be seen that for quite substantial regions of parameter-space the fixed point value $x = 1$ is approached quite closely at M_U .

In Fig.3 we present a similar plot showing the approach of y to the fixed point; in this case also there are sizeable regions such that $|y - \frac{1}{3}|$ is small at M_U . Here we have taken $\lambda_i(M_P) = \rho(M_P) = 4.9$, which is close to the limit for perturbative believability. It is interesting that if we take larger values of $\lambda_i(M_P), \rho(M_P)$ then although we are then starting further from $P = \frac{1}{3}Q$, the soft couplings approach the fixed point more quickly; but as we increase the dimensionless couplings, perturbation theory becomes less trustworthy, of course.

[†] It is also possible that above an intermediate compactification scale M_c the effective theory contains towers of Kaluza–Klein states; these may actually improve the rate of approach to the fixed point[5].

Fig.1: Evolution of $P - \frac{1}{3}Q$ from M_P to M_U , for various input values of λ_i, ρ at M_P . All the curves correspond to $\lambda_i(M_P) = 5\rho(M_P)$. The x-axis is $\log_{10}(M_P/\mu)$.

Fig.2: Contour plot showing input values of x and $\tilde{\lambda}_i = \tilde{\rho}$ at M_P that lead to $|x - 1| < 0.1, 0.2, 0.5$ at M_U .

Fig.3: Contour plot showing input values of x and y at M_P that lead to $|y - \frac{1}{3}| < 0.1, 0.2, 0.5$ at M_U .

It is also interesting to explore what happens if “weak” universality does not hold at the Planck mass.

Fig.4: Evolution of the ratio of A_1 to $A_\rho (\equiv A_4)$ from M_P to M_U , for various input values at M_P . The x -axis is $\log_{10}(M_P/\mu)$.

In Fig.4 we show how the A -parameters evolve if we assume that $A_1 = A_2 = A_3 = 0.1 \neq A_\rho$ at M_P . Shown is the evolution of the ratio A_1/A_ρ from M_P to M_U , for various starting values. All the curves correspond to $\lambda_i = 4.9$ and $\rho = 0.98$. Even though, as discussed above, Eq. (9a) is not IR attractive in this case, it is still approached quite well at M_U .

The soft scalar masses exhibit similar behaviour. In Fig.5 we show how a non-universal choice of masses at M_P converges quite rapidly to the fixed point as we approach M_U . We have taken $\tilde{A}_i = 0.1$, $\tilde{A}_\rho = 0.02$, $m_X = m_Y = M \neq m_Z$, and used various input values for $(m_X/m_Z)^2$.

Fig.5: Evolution of the ratio $(m_X/m_Z)^2$ from M_P to M_U , for various input values at M_P . The x-axis is $\log_{10}(M_P/\mu)$.

In all the above results it is assumed that there is no dependence on the flavour indices $\alpha, \beta \dots$. We have explored various specific forms for flavour dependence, and found that whether IR stability is maintained depends on the flavour structure. Before an exhaustive analysis of the possibilities it may be appropriate to construct a more realistic theory.

In this model there are no gauge invariant b_{ij} terms. For models with such terms, it is our experience that it is typically difficult to arrange for both IR stability of Eq. (9c) and for rapid approach to the fixed point. From this point of view, the relevant boundary conditions for low energy phenomenology may be $-A = \sqrt{3}m = M$, with B a free parameter, rather than $B = 2M/\sqrt{3}$ as in Refs. [3], [4]. Precisely these boundary conditions were in

fact explored in Ref. [11]. We would argue that they are relevant without the need of the special assumptions necessary to derive them from string theory.

We conclude by a reiteration of our basic philosophy, which transcends the details of the toy models we have presented. If universal scalar masses and cubic couplings at M_U are to be an infra-red phenomenon, they will necessarily be of the specific form shown in Eq. (9a) and (9b), and this can only be achieved in the class of theories which can satisfy $P = \frac{1}{3}Q$. This results in a substantial sharpening of the predictions for the superparticle mass spectrum at low energies[11]; even more so[3] if we suppose that Eq. (9c) is also approached.

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References

- [1] N. Polonsky and A. Pomarol, Phys. Rev. Lett. **73** (1994) 2292.
- [2] I. Jack and D.R.T. Jones, Phys. Lett. **B349** (1995) 294.
- [3] I. Jack, D.R.T. Jones and K.L. Roberts, LTH 347 (hep-ph/9505242)
- [4] D.R.T. Jones, LTH 350 (hep-ph/9506206).
- [5] M. Lanzagorta and G.G. Ross, Phys. Lett. **B349** (1995) 319.
- [6] D.R.T. Jones, L. Mezincescu and Y.-P. Yao, Phys. Lett. **B148** (1984) 317.
- [7] I. Jack and D.R.T. Jones, Phys. Lett. **B333** (1994) 372.
- [8] M.B. Einhorn, G. Goldberg and E. Rabinovici, Nucl. Phys. **B256** (1985) 499.
- [9] B. Pendleton and G.G. Ross, Phys. Lett. **B98** (1981) 291.
- [10] C.T. Hill, Phys. Rev. **D24** (1981) 691;
C.T. Hill, C.N. Leung and S. Rao, Nucl. Phys. **B262** (1985) 517.
- [11] R. Barbieri, J. Louis and M. Moretti, Phys. Lett. **B312** (1993) 451; erratum—*ibid* 316 (1993) 632;
J.L. Lopez, D.V. Nanopoulos and A. Zichichi, Phys. Lett. **B319** (1993) 451